

## Elastic full waveform inversion of microseismic data for location and source mechanism

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### Summary

We developed an elastic full waveform inversion method for determining microseismic event location and source focal mechanism simultaneously. With given P- and S-velocity models, the method inverts three component data and iteratively updates event location and source moment tensor solutions. Unlike inverting seismic data from controlled sources, we must infer origin time of microseismic events during inversion in order to match waveform properly. This is solved by calculating cross correlation of the input and synthetic waveform envelopes and extracting time shifts during each inversion iteration. Because the values of location and moment tensor parameters are not in the same order, we apply a scaling factor to adjust the updates so that they both can be resolved. Numerical experiments and real data applications suggest that fitting full waveform data leads to reliable solutions.

### Introduction

Accurate microseismic event location provides the information of the fracture geometry and growth. The source mechanism described by six components of moment tensor is another piece of information to better understand the fracturing process because it represents the strain near the located source (Baig and Urbancic, 2010). Using full waveform matching scheme is a common approach to infer the source mechanism in natural earthquake field (Zhao and Helmberger, 1994; Zhu and Helmberger, 1996). Song and Toksöz (2011) use full waveform to invert for the full moment tensor with six components using an initial event location as input. In addition, a cascaded grid search around the initial location is applied to generate a Green's function library. The inversion for an accurate source mechanism relies on the right event location.

Most of event location techniques only utilize the traveltimes information of the waveform phases (Zhang and Zhang, 2013; Eaton et al., 2011). The P- and S-wave moveouts are used to yield a 2D location with 1D velocity model. The event azimuth can be derived from the polarization information of the P wave (Magotra, 1989). For the migration-based location, an advantage of these methods is that they do not require picking the P- and S-wave arrival times (Zhang and Zhang, 2013; Eaton et al., 2011). For our full waveform inversion scheme, we do not need to pick arrival times either but we need a guess for the initial event location. Our full waveform inversion will

yield the final location with azimuth information and the source mechanism.

Microseismic data may contain many seismic phases (P and S direct waves, P and S converted waves and so on), we may simultaneously fit not only the direct wave but also the other seismic phases through full waveform inversion. Our synthetic examples show the information carried by the other phase information is also crucial for the inversion of location and source mechanism. In our inversion method, we apply General Reflection and Transmission Method (GRTM) for synthetic data calculation (Zhang et al., 2003). And cross correlation between the synthetic and the real waveform envelopes is applied to infer the origin time of the event during each iteration of the inversion.

### Theory

We assume the velocity model and source time function are known and the source location and source mechanism are the unknowns that we try to solve for. Because the values of the location and moment tensor may not be in the same order, we apply a scaling factor to normalize the amplitude of the input data before starting the inversion. In this way, the inverted moment tensor is corresponding to the scaled input data. However, the exact moment tensor with respect to the original input data can be derived if we multiply inverted moment tensor with the same factor as the normalization, because of the linear relationship between the displacement and the moment tensor. We need a nonlinear least squares iteration scheme to perform the full waveform inversion for location and source mechanism simultaneously. The objective function that we adopt is as following:

$$\begin{aligned}\psi(m) &= \sum_{i,t} \|d_{obs,i}(t) - u_i(m, t, \Delta t)\|^2 \\ f(m, \tau) &= \sum_i \int \tilde{d}_{obs,i}(t - \tau) \tilde{u}_i(m, t, 0) dt \\ \Delta t(m) &= \max_{\tau} f(m, \tau) \\ m &= (s_x, s_y, s_z, M_{xx}, M_{yy}, M_{zz}, M_{xy}, M_{xz}, M_{yz}),\end{aligned}\quad (1)$$

where  $i$  represents each trace in the microseismic data (including the three components of displacement), so the total number of traces is three times the number of receivers.  $d_{obs,i}$  and  $u_i$  are the observed and synthetic waveforms.  $\tilde{d}_{obs,i}$  and  $\tilde{u}_i$  represents the corresponding observed and synthetic waveform envelopes. The details for the calculation of trace envelope are given in Luo and

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Wu (2013). The model includes the source location and the six elements of moment tensor. Because we do not have the origin time for the microseismic data, we need to line up the synthetic data by cross correlation between real and synthetic waveform envelopes so that the subtraction is between the right phases.  $u_i(m, t, \Delta t)$  denotes the synthetic trace with time shift  $\Delta t$ .  $\Delta t$  is corresponding to the origin time and can be estimated by picking the peak of the cross correlation of the envelopes.

The analytical solutions are available for the derivative of the waveform with respect to the model parameters. These analytical solutions facilitate our inversion. The derivative of the displacement with respect to the moment tensor is the convolution between the gradient of the Green's function and the source time function according to representation theory (Aki and Richards, 1980):

$$\frac{\partial u_n}{\partial M_{pq}} = G_{np,q} * s(t) \quad (2)$$

where  $u_n$ ,  $G_{np,q}$  and  $M_{pq}$  represent the displacement, the derivative of the Green's function and moment tensor respectively.  $s(t)$  is the source time function. For simplification we just use the numerical method to calculate the derivative of the displacement with respect to the source location. We only need one forward modeling operation to get the derivative of the displacement with respect to the moment tensor. We can obtain the model increment of the objective function in Equation (1) using nonlinear least squares method:

$$\sum_{i,t} \frac{\partial u_i(m,t,\Delta t)^T}{\partial m} \Delta d(t) = \sum_{i,t} \frac{\partial u_i(m,t,\Delta t)^T}{\partial m} \frac{\partial u_i(m,t,\Delta t)}{\partial m} \Delta m$$

$$\frac{\partial u_i(m,t,\Delta t(m))}{\partial m} = \frac{\partial u_i(m,t,\tau)}{\partial m} \Big|_{\tau=\Delta t} + \frac{\partial u_i(m,t,\Delta t)}{\partial \Delta t} \frac{\partial \Delta t}{\partial m}$$

$$\frac{\partial \Delta t}{\partial m} = \frac{\partial \dot{f}_{\Delta t}}{\partial \dot{f}_{\Delta t}}, \quad \dot{f}_{\Delta t} = \sum_{\tau} \int \dot{d}_{obs,i}(t - \Delta t) \dot{u}_i(m,t,0) dt, \quad (3)$$

where  $\dot{f}_{\Delta t}$  is the derivative of the function  $f(m, \tau)$  with respect to time  $\tau$  at  $\tau = \Delta t$ . The derivative of the displacement with respect to the location is also related to  $\Delta t$ . The derivative of  $\Delta t$  with respect to the model can be calculated from the implicit function of the cross correlation formula.

### Synthetic examples

We test the algorithm in two synthetic examples for different velocity models. The first is a velocity model derived from a field data (Figure 1b) and the second is designed with two simple layers (Figure 2b) so that the

waveform is not as complex as in the first example. For both examples, eight receivers are deployed from 3885m to 3955m in depth with depth interval of 10m.

Figure 1 shows the true model for the source location and mechanism, and the velocity input for example 1. The horizontal distance from the true location to the receiver array is 300m. To study the source mechanism more conveniently, we set the y coordinate to be zero. We use a double couple source as the true source mechanism, but our method tries to recover the full moment tensor of the source. Figure 3a shows the azimuth of the initial location is far from the true location and the depth of the initial location is also different from the true location. The initial source mechanism is not a double couple and the magnitude is also larger than the true magnitude. Figure 3b shows the result of the inversion. We can get an accurate event location that is almost the same as the one of the true model. In addition, not only the direct waves but also the waveform phases followed by the direct waves can be matched very well. However, the inverted source mechanism solution is still not a double couple, even though the final synthetic waveform and the location are accurate. Table 1 contains the values of the inverted location and source mechanism parameters. All the parameters can be recovered properly except  $M_{yy}$ .  $M_{yy}$  is underdetermined using measurements from a single well unless the near field waveform is strong enough. This conclusion is consistent with the result by Song and Toksöz (2011).

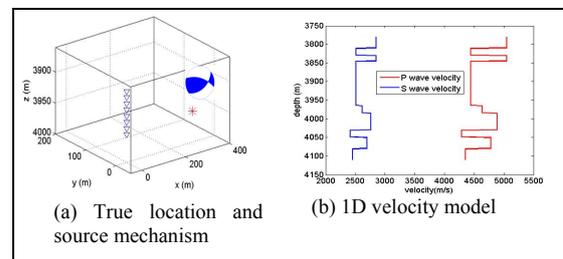


Figure 1: True model and velocity for example 1. The true source location is (300,0,3950).

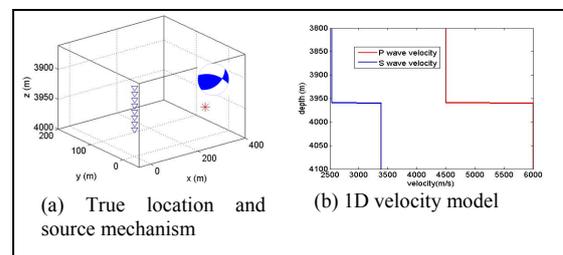
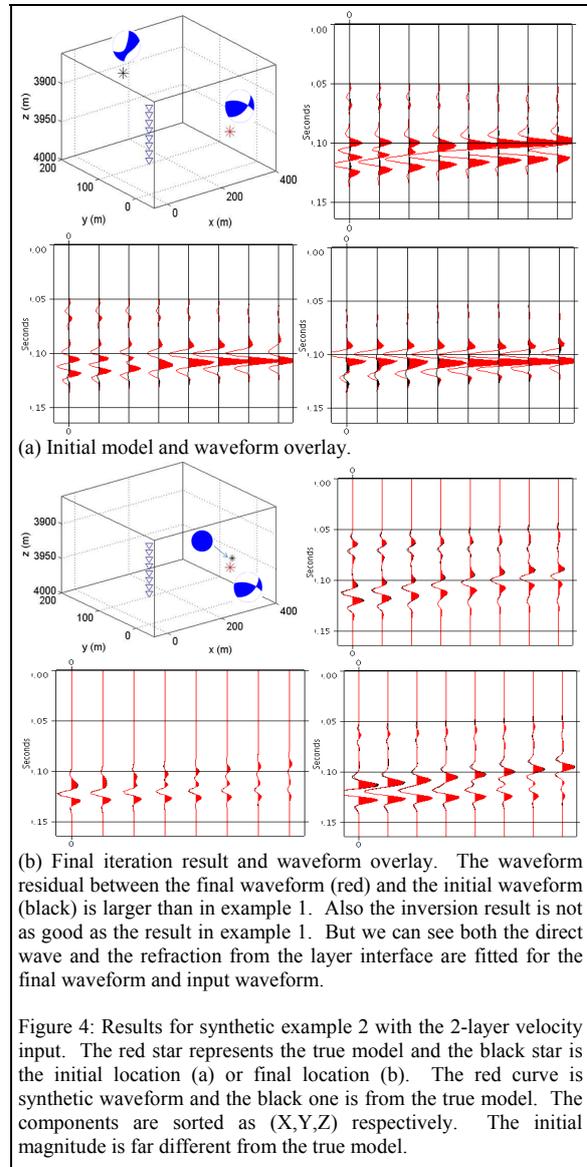
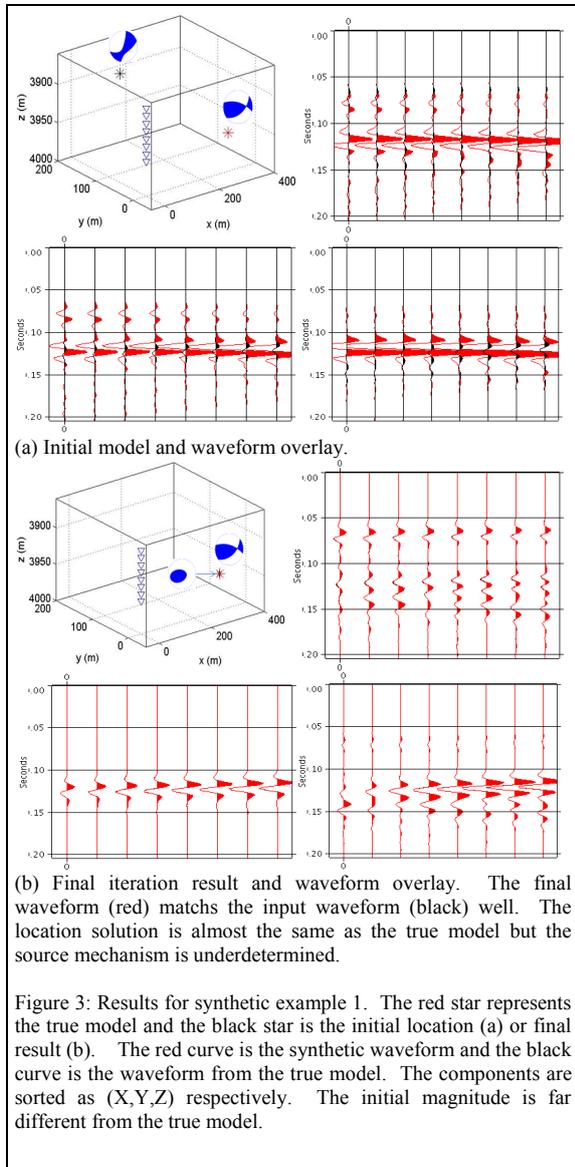


Figure 2: True model and velocity for example 2. The only input that is different from example 1 is the simpler velocity model.

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	$s_x$	$s_y$	$s_z$	$M_{xx}$	$M_{yy}$	$M_{zz}$	$M_{xy}$	$M_{xz}$	$M_{yz}$
True model	300m	0m	3950m	-0.6997	0.1544	0.5453	0.2015	-0.2478	-0.2954
Solution for example 1	300.0m	0.8m	3950.0m	-0.7194	-0.1571	0.5253	0.1997	-0.2469	-0.3035
Solution for example 2	320.2m	8.7m	3941.2m	0.0246	1.4648	1.4788	0.0907	-0.1115	-0.4271

Table 1: Inversion results for synthetic examples.

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Figure 2 shows the initial model and geometry for example 2 are the same as the ones for example 1. The only difference between the two examples is the velocity model. Figure 4a shows that the synthetic waveform calculated by initial model parameters is very different from the input data. Figure 4b shows the error of the event location solution is about 24m. The source mechanism is entirely different from the true parameters. Table 1 shows the values of the inversion result. However, we can also see the final synthetic waveform matches the input waveform although the waveform residual is slightly larger than the one of example 1. The result shows that not only the direct wave is recovered but also the refraction from the velocity interface is, too. The waveform phase of example 2 is much simpler than the one of example 1. However, the solution of the example 2 is not as good as the one of example 1 because the other kinds of complex waveform phases also contain the information of the location and source mechanism. The incorrect event location may also affect the source mechanism solution because of the interdependency of the source mechanism and location.

### Field data example

We test the algorithm in a field data example, see Figure 5. The receiver array is located from 1120m to 1285m in depth with interval of 15m, see Figure 5a. Figure 5b shows the velocity model from sonic logging. The initial location is estimated to be (-70m, 0m, 1421m) and the source mechanism is randomly chosen as the same with the one of synthetic example section. Figure 5c shows the overlay of initial synthetic waveform and the field data waveform after normalizing. We can see the phase of y component is completely reversed in the synthetic waveform compared with real data. Figure 5d shows that the final synthetic

waveform well matches the real data and P wave also can be fitted. The result shows that the source mechanism of the final solution is like a strike-slip fault. The strike direction is almost in the plane formed by receiver array and source. The location of the inversion result is (-86.1m, 2.4m, 1426.2m).

### Conclusions

We develop a method to perform full waveform inversion for event location and full moment tensor. And we borrow the envelope concept to align the main energy between the input waveform and the synthetic waveform during the inversion. Synthetic examples show that we may fit all the waveform phases simultaneously using the inversion method. The other waveform phases followed by the direct waves also contain important information about event location and source mechanism. For full waveform inversion in the case of measurements from single well, we may not get the right solution for the source mechanism even if the final synthetic waveform well matches the input waveform. Therefore, the extra regularization for the source mechanism may be required. Because the analytical solution of 1D VTI media is available, this method may be applied for inversion of source location and source mechanism in 1D VTI media. Area of future research is to update the velocity model, while inverting for the source parameters.

### Acknowledgements

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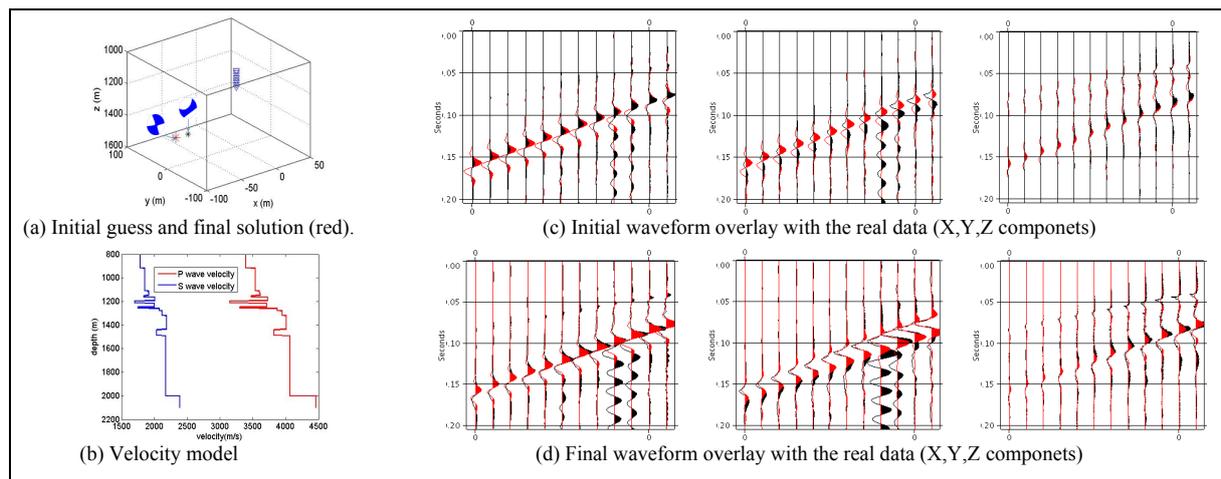


Figure 5: Real data example. The initial guess and the final solution are as (a) shows. The final waveform (red) well matches the real data (black) well.

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#### EDITED REFERENCES

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