Residual statics solution by L1 regularized inversion in common offset domain

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Summary

Residual statics estimation is an important step in seismic data processing. We develop a methodology for determining residual statics with common offset gathers (COG). The methodology is divided into two steps: 1) inverting the total residual statics of each trace in COG; 2) decomposing the total residual statics into the shot and the receiver residual statics. In the first step, the total residual statics can be inverted from equations derived from the refraction traveltimes in the common offset domain constrained by L1 norm regularization. In the second part, the total residual statics of each trace is decomposed into shot and receiver residual statics with zeroth-order Tikhonov regularization. The synthetic examples show that the method can help determine large magnitude residual statics and mitigate the effect of picking errors. The method is applied to real data and the results show improved quality of CMP stacks.

Introduction

Making statics correction is very important in land and shallow marine data processing. The long wavelength statics can be calculated from the velocity model of the near surface. However, residual statics may also be crucial to improve the quality of the stacking image. It is usually derived from refraction or reflection data. Residual statics is often constrained to small values.

Residual statics methods usually apply cross-correlation calculation to reflection data (Taner et al., 1974; Ronen and Claerbout, 1985). The time difference, which is obtained by cross-correlation between super pilot trace and individual trace, is decomposed into shot and receiver residual statics, residual NMO component, and structure term (Taner et al., 1974). The statics result may be sensitive to the selections of the maximum allowable shift and correlation window if the value of the residual statics is large. Using refraction or first arrival traveltimes to derive residual statics for reflection data is an indirect approach. Taner et al. (2005) calculate the statics by decomposing the first break time into shot and receiver delay time, shot and receiver residual statics, and a summation of traveltime through the related cells of refractor interface. Zhu and Luo (2004) decompose the traveltime difference between picks and smoothed curves into shot and receiver residual statics.

In this study, we apply an iteratively reweighted least squares (IRLS) algorithm to optimize the residual statics equations derived from the common offset gather with L1 norm regularization (Scales et al., 1988). We modify the IRLS process by considering lateral velocity variations and undulation of refractor for estimating large residual statics. Our algorithm is demonstrated with synthetics and real data examples.

Method

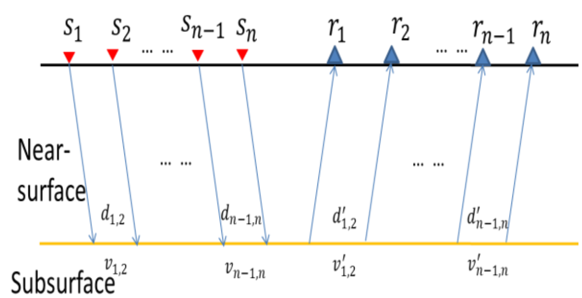


Figure 1: Schematic illustration of refraction raypath in a common offset gather. s_n and r_n represent the n_{th} shot and the n_{th} receiver, respectively. r_n detects the signal form s_n. Shot-receiver offset for each trace is same. Items with d and items with v represent the distance and the velocity, respectively, for corresponding refractor cells.

The traveltime along the raypath of each shot-receiver pair in the common offset domain (Figure 1) can be decomposed as following:

$$\mathrm{T} = T_{basic} + res \tag{1}$$

where T_{basic} is the traveltime without residual statics. res is the total residual statics, which should be eliminated from the traveltime T. we can calculate the shot and receiver statics through two steps: 1) inversion for total residual statics res for each trace without considering picking errors; 2) decomposition into shot residual statics and receiver residual statics with surface-consistent assumption.

Equation for estimating the total residual statics is based on traveltime difference between adjacent traces in the common offset gather and defined as:

$$\begin{aligned} \Delta T_{12} &= T_2 - T_1 = res\,(2) - res(1) + \Delta G_{1,2} \\ \Delta T_{23} &= T_3 - T_2 = res\,(3) - res(2) + \Delta G_{2,3} \\ &\dots\dots \\ \Delta T_{k-1,k} &= res\,(k) - res(k-1) + \Delta G_{k-1,k} \end{aligned} \tag{2}$$

where *res(k)* is the total residual statics for the k_{th} trace, which is related to the k_{th} shot-receiver pair. $\Delta G_{k-1,k}$ represents the difference between T_{basic} for the k_{th} trace and the $k-1_{th}$ trace and is caused by undulation and lateral velocity variations of refractor interface. For the case in Figure 1, we define $\Delta G_{k-1,k} = \frac{d'_{k-1,k}}{v'_{k-1,k}} - \frac{d_{k-1,k}}{v_{k-1,k}}$.

We refer to the least 2-norm solution as a preliminary one for equation 2:

$$\begin{gathered}\text{Minimize } ||Wm||_2 \\ \text{Subject to } Gm = \Delta T\end{gathered} \tag{3}$$

where m is the vector [$res(1)$, $res(2)$,..., $res\ (k)$, $\Delta G_{1,2}$,..., $\Delta G_{k-1,k}$]; G is the system matrix of equation 2; ΔT is the traveltime difference vector [ΔT_{12} , ΔT_{23} ,..., $\Delta T_{k-1,k}$]; W is an identity matrix for the least 2-norm solution. For estimating large statics, we refer to $\frac{1}{\|m\|}$, m in which is the preliminary solution, as diagonal elements of W and solve equation 3 iteratively. For large absolute value in preliminary solution, there is a small $\frac{1}{\|m\|}$, which leads to a large value in the updated solution. The above process is close to IRLS algorithm for the L1 regularized problem:

$$\begin{gathered}\text{Minimize } ||m||_1 \\ \text{Subject to } Gm = \Delta T\end{gathered} \tag{4}$$

The procedure is defined as:

$$\begin{aligned}&\text{Repeat } \{ \\ &\qquad W_{i,i} = \frac{1}{\|m\|} \\ &\text{m} = W^{-1}G^T(GW^{-1}G^T)^{-1}\Delta T \\ &\qquad \}\text{ until convergence}\end{aligned}$$

In our application, if we fully apply IRLS algorithm, then we cannot estimate large statics effectively. We modify the IRLS process for the W. The last $\text{k}-1$ diagonal elements for W are related to items ΔG, which are relatively small values compared to residual statics (typically $\pm$10 ms) with raypath and velocity along the refractor for two adjacent traces. We select a unified relatively large value for the last $\text{k}-1$ diagonal elements and estimate large statics in synthetic tests and real example. We can adjust the last $\text{k}-1$ elements according to the picked refracted arrivals in one COG.

The above solution is decomposed into shot and receiver residual statics through linear inversion with Tikhonov regularization (Tikhonov and Arsenin, 1977). The equation for decomposition is defined as:

$$res\ (i,j) = s_i + r_j \tag{5}$$

where $res\ (i,j)$ is the total residual statics, the sum of the i_{th} shot residual statics and the j_{th} receiver residual statics, which are represented by s_i and r_j, respectively. The inverse process is defined as:

$$\begin{gathered}\text{Minimize } (\|Dr - res\|_2^2 + \alpha^2\|r\|_2^2) \\ \text{Minimize } \left\|\begin{bmatrix} D \\ \alpha I\end{bmatrix} r - \begin{bmatrix} res \\ 0\end{bmatrix}\right\|_2^2 \\ \text{r} = (D^TD + \alpha^2 I)^{-1}D^T res\end{gathered} \tag{6}$$

where solution r consists of s_i and r_j . res represents $res\ (i,j)$. D is the system matrix of equation 5. The regularization parameter α is determined by L-curve method (Hansen, 1992).

Synthetic Data Tests

We test the algorithm in a model with undulating refractor interface. Figure 2 illustrates a simple velocity model with two layers and acquisition geometry. For each shot, we have a recording template of 300 receivers (150 receivers on each side). A total of 500 shots are recorded with a shot interval of 20 m and a receiver interval of 20 m.

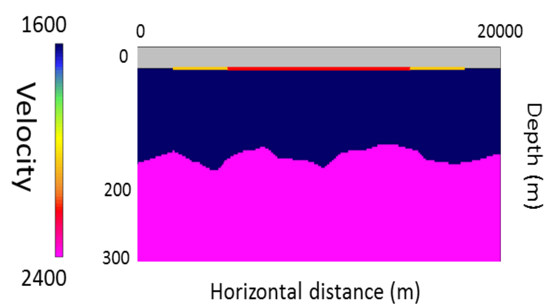


Figure 2: Undulating refractor model

We directly produce random shot and receiver residual statics according to the surface-consistent assumption between -5 ms and 5 ms and set large magnitude residual statics for parts of shots and receivers with the maximum value of 25 ms. Figure 3 shows the synthetic shot and receiver residual statics. For noise-free data, the results are illustrated in Figure 4 (a) and (b). In the field data processing, there may be picking errors in the picked first arrivals. We refer the picking error as noise for the residual statics correction problem. We add independent normal distributed noise with a standard deviation of 8 ms to synthetic first breaks. We determine the last $\text{k}-1$ diagonal elements for W by analyzing the COG data with a specific offset. Results are illustrated in Figure 4. The red line represents the common-offset traveltime curve with residual statics applied. Figure 4 (a) and (c) show the results derived from

IRLS algorithm for noise-free data and noisy data, respectively. The COG traveltime curves with statics correction applied still contain large statics. We apply the algorithm using the data with offset range of 1500-3000 m and employ $W_{i,i} = 20$ for the last k-1 diagonal elements of W. Results are demonstrated in Figure 4 (b) and (d). Large statics estimation is improved after three iterations.

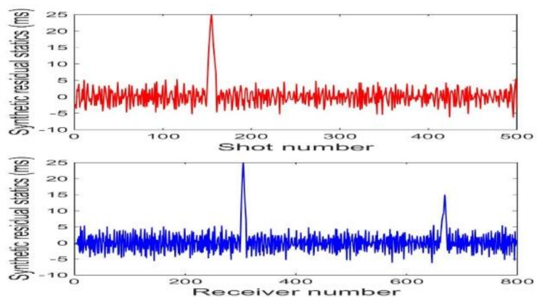
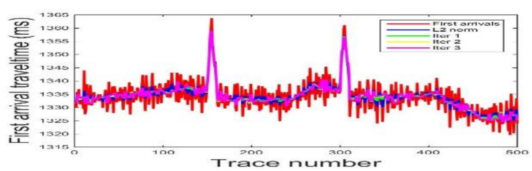
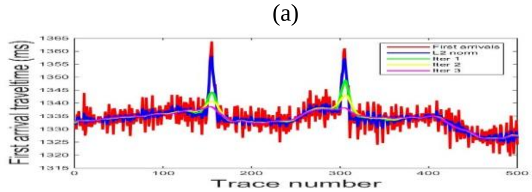


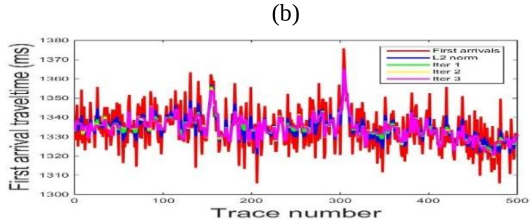
Figure 3: Synthetic shot and receiver residual statics



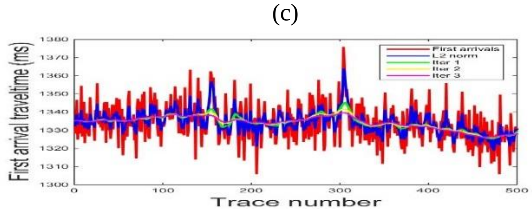
(a)



(b)



(c)



(d)

Figure 4: (a) results with full IRLS algorithm for data without noise. The red line is synthetic first arrivals. The blue curve is the result of least 2-norm solution. Other curves are results with IRLS algorithm. (b) Results with modified IRLS process for data without noise. (c) Results with full IRLS algorithm for data with noise. (d) Results with modified IRLS process for data with noise.

After decomposition for total residual statics, the inverted and the true residual statics are shown in Figure 5. Fang and Zhang (2014) characterize the recovering degree of the synthetic model using resolvability, which is defined as:

$$R = \frac{\sum_{j=1}^{m}(Dn_{tj}+Dn_{rj})^2}{2\sum_{j=1}^{m}(Dn_{tj}^{\,2}+Dn_{rj}^{\,2})} \tag{7}$$

where Dn_{tj} and Dn_{rj} are the true and recovered model parameter at the j_{th} node inside the model region consisting m nodes. In the test, we set m=11. For noisy data, the recovering degree is larger than 0.85 mostly.

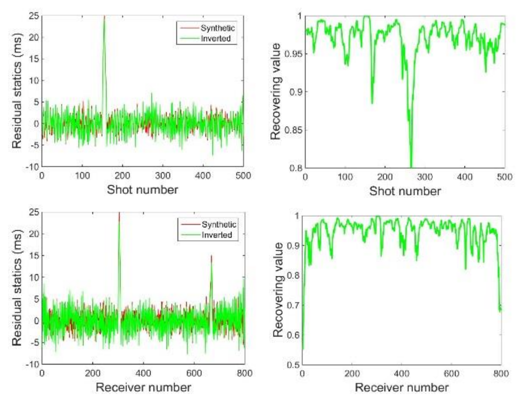


Figure 5: Recovered residual statics and resolvability with noise. The left panel is the comparison between true synthetic statics and inverted statics. The right panel is the calculated recovering degree for residual statics inversion.

Field Data Test

We apply the methodology to a real 2D dataset with 485 shots. Figure 6 shows the original picked first arrival curve (red) and traveltime curves with different statics solutions in the process of estimation for the total residual statics. We apply the algorithm using the data with offset range of 1800-2400 meters and employ $W_{i,i} = 20$ for the last k-1 diagonal elements of W.

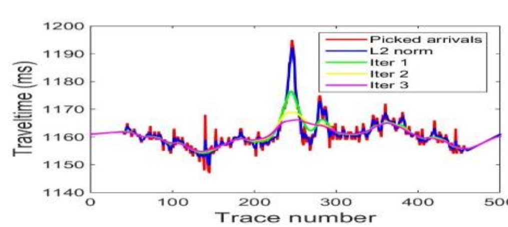


Figure 6: Picked first arrivals (red) and results with different number of iteration. The blue curve is the COG traveltimes with

least 2-norm solution. Other curves are the COG traveltimes obtained in modified IRLS process for residual statics inversion.

Figure 7 shows a shot gather before and after the application of residual statics corrections. The distortion in reflection corresponds to the distortion in the first arrival, thus refraction statics should be effective. The comparison of NMO corrected CMP gathers before and after statics applied in Figure 8 indicates that the statics is indeed effective for reflection. Figure 9 shows a common offset gather before and after statics applied. Figure 10 shows the CMP reflection stacking sections before and after residual statics corrections applied. The continuity and quality have been improved considerably in the marked area as shown in Figure 10.

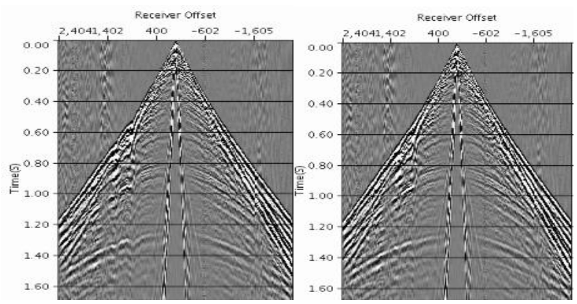


Figure 7: A 2D shot gather before (left) and after (right) the application of residual statics correction.

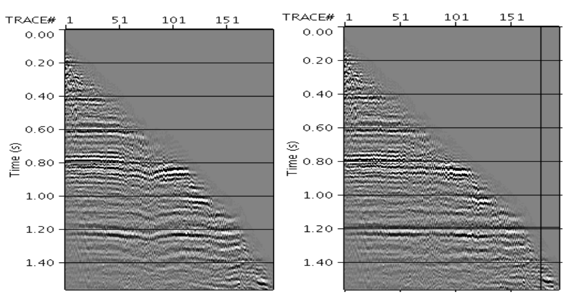
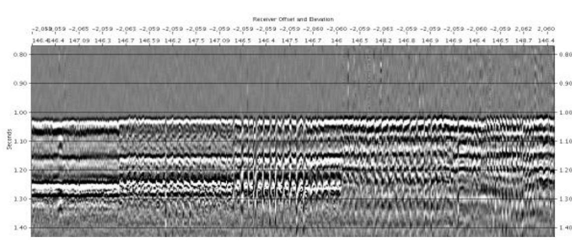


Figure 8: NMO corrected CMP gathers before (left) and after (right) the application of residual statics correction.



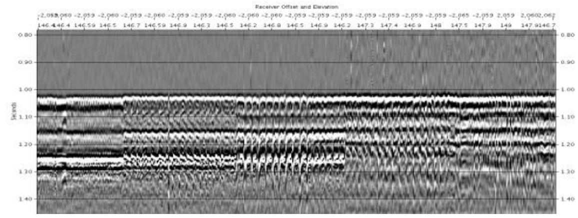


Figure 9: Common offset gather before (upper) and after (under) the application of residual statics.

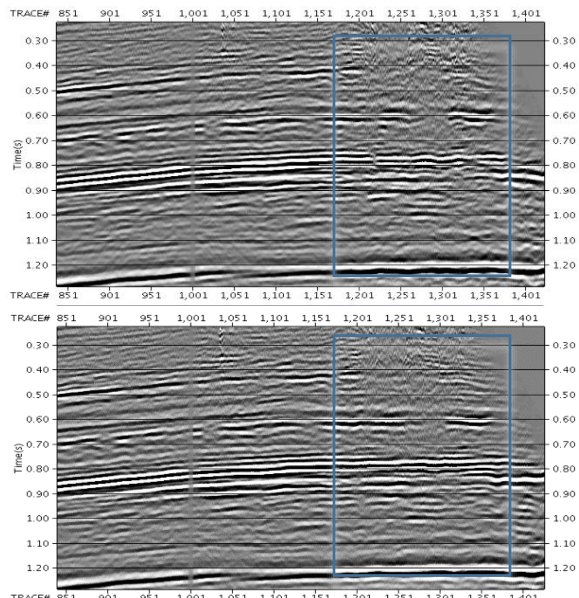


Figure 10: 2D stacked section before (upper) and after (under) the application of residual statics.

Conclusions

We have proposed a methodology for solving large residual statics correction problem: focusing on the characters of first arrivals between adjacent traces in common offset gathers, modeling the residual statics problem and solving it. We modify the IRLS process for L1 regularized inversion to estimate large residual statics. The method can determine large magnitude residual statics in synthetic tests and improve the quality of results in processing real dataset.

Acknowledgement

We appreciate the support from GeoTomo for allowing us to use TomoPlus software to perform this study.